

The Armington Model

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Abstract

Armington (1969) made the assumption that products traded internationally are differentiated on the basis of their country of origin. Models based on this assumption are dominant among cge models but their general properties are unknown. This paper explores analytically the properties of such models. Propositions are derived relating to the Law of One Price, comparative advantage, factor price equalisation, the nature of intra-industry trade and gains from trade. It is shown that the Armington Assumption leads to properties that are fundamentally different than those of the Heckscher-Ohlin and other models based on homogeneous products.

1. Introduction

In a well-known paper, Armington (1969) introduced the assumption that final products traded internationally are differentiated on the basis of the location of production. He assumed that in any one country each industry produces only one product and that this product is distinct from the product of the same industry from any other country. For simplicity, he assumed there is only one consumer in each country. In the eyes of this consumer, the products of one industry which originate in the different countries are a group of close substitutes. They form a product group which is separable in the consumer's utility function.

This Armington assumption of nationally differentiated products has been widely adopted in global computable general equilibrium (cge) models to define demands for domestically produced and imported goods. When imports originate in more than one foreign country, these models usually assume that imports from different countries are differentiated from each other and form a group that is separable from the domestically produced product. Cge models which incorporate traded intermediate inputs also almost invariably assume that the domestically produced and the imported inputs used in one industry are imperfect but close substitutes, nested in a production function for the industry output. Together these assumptions on the demand side of a trade model give a lot of structure to the trade model.

The Armington structure was introduced in cge models to overcome problems that arose in early cge modelling efforts. It accommodates 'cross-hauling', a phenomenon commonly observed in bilateral trade statistics, that is, a country appears to export and import the same goods simultaneously. This phenomenon cannot be explained in traditional trade models with homogeneous goods.¹ The Armington structure also overcomes the problem in a Heckscher-Ohlin model with more goods than factors that countries tend to specialise in only a few of the goods produced. (Choi, 2003 and Deardorff, 2005 gives a modern treatment of this feature of the Heckscher-Ohlin model). Thus, the Armington assumption of product differentiation and imperfect substitution makes the existing trade statistics immediately usable for global trade models.

In a survey of applied general equilibrium modelling, Shoven and Whalley (1984, p. 1046) commented:

‘(B)ecause of the difficulties in accommodating a wide range of empirical phenomena in model building, there is often a tendency to depart from the essential structure and graft on *ad hoc* portions of the model not rooted in traditional theory... Unfortunately, the problem is, the models that make major departures from known theoretical structure can become difficult to interpret.’

The introduction of Armington substitution is a departure from the assumption of perfect substitution that underlies traditional trade theory. Although Armington models have been widely used for many years, their analytical properties have not been developed. We seek to characterise the general equilibrium of an Armington model in a general way. In fact, we find that the Armington structure does change fundamentally the properties of a trade model.

Section 2 specifies the Armington model. Section 3 develops the properties of the competitive equilibrium of this model for both free and tariff-distorted trade situations. These are compared with the properties of standard Neoclassical models, especially the Heckscher-Ohlin model. In the light of the findings, Section 4 offers some comments on the interpretation of results from cge models that employ the Armington Assumption.

2. The Armington model

Demand side

Armington (1969) considered a world in which there are m countries and n goods. Each good is a “type” or “kind” of good, one variant of which may be produced in each country. Thus the goods are differentiated by country of origin. Consequently there are $m \times n$ “products” or commodities over which consumers have defined preferences. The number of goods and therefore the total number of products in the model is fixed.

When the consumption of a good is differentiated in this way, it is necessary to have three subscripts attached to quantities and prices of products in order to identify the good itself, the country of origin of the good and the country in which the good is consumed. Let $i = 1, \dots, n$ denote the type of good, j

$i = 1, \dots, m$ the country of origin of the product variety and $k = 1, \dots, m$ the country in which it is located.

Thus the first two subscripts identify the product and the third the country in which it is consumed.

Then x_{ijk} is the consumption of a product from the i 'th group when the product originates from country j and the consumer is located in country k , and p_{ijk} is the price of this product in country k .

In each country there is only one household. With Armington differentiation the utility function of the household of one country, country k , takes the form

$$U_k = U(x_{11k}, \dots, x_{1mk}, \dots, x_{i1k}, \dots, x_{imk}, \dots, x_{n1k}, \dots, x_{nmk})$$

$$= V(v^1(x_{1k}), \dots, v^i(x_{ik}), \dots, v^n(x_{nk})) \quad \forall k \quad (1)$$

where $x_{ik} = (x_{i1k}, \dots, x_{imk})$ is the set of products of type i originating in the different countries and consumed in country k . It is assumed that the utility function is identical across countries, $U(\cdot)$. Thus, this utility function is assumed to be weakly separable in the types of goods. The sub-functions $v^i(x_{ik})$ are indices of the consumption in the country of each type of good. Moreover, these sub-functions are linearly homogenous. Thus, the utility function is homogeneously separable. This has the important consequence that the household allocates its budget in two stages, first at the top stage or level among the groups of products and then at the bottom stage or level among the products within a group.

Armington adopted a CES form for the functions at the second level: viz.

$$v^i(x_{ik}) = [\alpha_{i1k}x_{i1k}^{-\rho_i} \dots + \alpha_{ijk}x_{ijk}^{-\rho_i} \dots + \alpha_{imk}x_{imk}^{-\rho_i}]^{-1/\rho_i} \quad \alpha_{ijk} \geq 0, \rho_i > -1; \forall i, k \quad (2)$$

When the technology for building multi-country cge models of the world economy became available, the Armington assumption of national product differentiation was incorporated in these models. The first cge model of a multi-country world economy was that of Shoven and Whalley (1974). Although they did not refer to Armington, they used the Armington differentiation in a single level CES function for each industry or type of good.

Within a few years several modelbuilders² adopted the Armington two-level structure and assumed that both the utility function at the top level, $V(\cdot)$, and all the sub-functions at the lower level, $v^i(\cdot)$,

are CES in form. This combination of constant upper- and lower-level elasticities of substitution means the utility function is analogous to the two-level CES production function introduced by Sato (1968). These models were the first models of the world economy to incorporate the Armington Assumption, and they were the first general equilibrium models of intra-industry trade.

In subsequent cge models, a typical Armington structure for final demand assumes a three-stage budgetary allocation procedure. Expenditure is first allocated among goods. Expenditure on each good is then allocated between domestic and imported varieties. (See, for example, the ORANI model outlined in Dixon, 1982, Section 14.) Finally, if more than one source of imports is acknowledged, expenditure on imports is allocated among competing national suppliers.³ The whole structure can be neatly captured in a set of nested functions:

$$U(x_k) = V(v^1(x_{1k}), \dots, v^i(x_{ik}), \dots, v^n(x_{nk})) \quad \forall k$$

where

$$v^i(x_{ik}) = f^i(g^{id}(x_{ikk}), g^{if}(x_{ifk}))$$

and, (3)

$$g^{if}(x_{ifk}) = g^{if}(x_{i1k}, \dots, x_{ijk}, \dots, x_{imk}) \quad j \neq k$$

Here, for convenience, *d* denotes the home or destination country, the country in which the consumer is located, and *f* a foreign source country. That is, the utility function is separable in types of goods, and the sub-functions for each type of good in turn are separable into the demand for the product originating in the home country and the demand for the products originating in foreign countries, and in turn the demands from foreign sources are themselves functions of the demand for each type of good supplied by each of the foreign countries. All of the functions v^i , f^i , and g^{if} are linearly homogeneous.

The first stage allocation does not involve Armington differentiation. It is based on an ordinary utility function with preferences allocated over different goods that are substitutable. The form of this utility function can be any well-behaved function used in consumer theory. In the Armington model, the ‘goods’ in this utility function are not actual goods but composites of domestic and imported goods,

which are aggregated from the second stage budgetary allocation. Only the second and third stage allocations are based on the Armington assumption of national product differentiation.

Some builders of cge models have assumed that the functions at all three levels are CES while others have assumed that they are CES at one or two levels. Some have assumed that, at some level, the functions are Leontief or Cobb-Douglas which are of course special cases of the CES function. The crucial assumption that makes a model an Armington model is that the utility function is separable, and the products in a group are differentiated by country of origin, in the manner of Equation (1).

Within this defining structure, there are a number of variations as in Equations (1) or (3).

The Armington form of national differentiation of products is only one possible form of product differentiation. The post-Armington literature distinguishes between horizontal and vertical differentiation. The former refers to different varieties of a product and the latter to different qualities of a product. Armington does not discuss the nature of his product differentiation but it resembles the “love of variety” type found in the models of Krugman (1979, 1980) which is a form of horizontal differentiation. Another form of horizontal differentiation is the “most preferred variety” found in the models of Lancaster (1979) and Helpman (1981). In both the Krugman and Lancaster-Helpman models, countries produce and export sets of differentiated products that are disjoint. Any of these forms of product differentiation can be introduced into a cge model. For example, the current version of the Michigan Model has adopted the “love of variety” form along with scale effects and monopolistic competition.

Moreover, in the Krugman and Lancaster-Helpman models each country produces and exports more than one product (=variety) and the number of varieties produced in each country is endogenously determined. By contrast, in the Armington model, national horizontal differentiation of products is a single product from each industry, exogenous and unexplained. In this model the range of products produced by each country and the range of products consumed cannot change when the competitive equilibrium of the model is disturbed for any reason. Hence, the beneficial effect of trade liberalisation through an increase in the number of varieties of products, both for final and intermediate use, cannot be captured in an Armington model.

Thus, the Armington model is a rather special form of horizontal product differentiation. Head and Reis (2001) find empirical support that bilateral trade between the US and Canada in differentiated products is of the Armington type rather than Krugman-Helpman type.

With the development of cge models that incorporate intermediate inputs as well as primary inputs, the assumption that final goods are differentiated by nationality has been extended to the demand for traded intermediate inputs. For example, this assumption has been adopted in the families of models developed by Whalley (1985), the Australian ORANI and Monash models (Dixon et al, 1982 and Dixon and Rimmer, 2002), and by GTAP (Hertel, 1997). As with the demand for final goods, it is assumed that foreign-produced intermediate inputs used in any industry may be nested in the industry production function.

The assumption that final and intermediate goods are differentiated by nationality and form separable groups in the utility and production functions will be called the Armington Assumption. The further assumption, also made by Armington, that the functions at one or more levels are CES, is not regarded as a part of the Armington Assumption. The assumption of CES does, however, have effects on the properties of the general equilibria.⁴

Armington (1969) specified only the demand side of the model.⁵ The term “Armington model” will be used to cover any model which makes the Armington Assumption on the demand side together with a standard Neoclassical specification of the supply side, that is, constant returns to scale and atomistic competition everywhere. This gives a family of models with a basic commonality in their structure.⁶ These models are now dominant in cge analysis of trade policy issues throughout the world and in gravity models.

Supply side

In each industry, there is a production function for each of the countries in the world economy:

$$y_{ij} = \psi_{ij}(e_{ij}, b_{ij}) \quad (4)$$

where e_{ij} is the vector of usage by industry i of the primary factors available in country j and b_{ij} is the vector of usage of intermediate inputs from different sources in the production of the good.

Intermediate inputs are, like the final outputs, differentiated by country of origin. It is not assumed, in general, that the technologies are identical across countries but it is assumed that there are constant returns to scale in all industries and all product and factor markets are perfectly competitive. All factors are in fixed supply but they may be specific or non-specific.

If required, the Armington structure may be imposed on the demand for intermediate inputs. The most common assumption is that there is a two-level demand function for intermediate inputs. At the top-level there is a demand in each industry for intermediate inputs supplied by the home country and by foreign countries collectively. At the second level, there is a function specifying the demand for foreign-supplied intermediate inputs from each of the foreign countries. (See, for example, Whalley, 1985, Dixon et al, 1982 and Hertel, 1997.)

Thus, the Armington model can be viewed as a standard Neoclassical model on the supply side with the addition of the Armington structure on the demand side. This viewpoint highlights the role played by the assumption of national differentiation of products.

This specification is sufficiently general to encompass a wide variety of models. It has dimensions $(m \times n) \times h \times m$ where $m \times n$ is the number of products, h is the number of primary (non-produced) factors and m is the number of countries in the world economy. Each of these numbers may be small or large and there is no restriction, in general, on the number of factors (h) in relation to the number of goods produced in any country (n). The number of industries and the number of countries have been small though the numbers have tended to increase in revisions of cge models. With national differentiation of the products of each industry, the number of products will be larger than the number of factors in almost all applications. Most commonly it is assumed that the number of factors is 2 (non-specific labour and capital) (for example, Whalley, 1985 and Dixon, 1982) or 3 (non-specific labour, capital and land) (for example, GTAP; see Hertel, 1997), as in the standard Heckscher-Ohlin model.

Competitive equilibrium

A competitive equilibrium for the world economy is one in which a set of prices for products and factors satisfies supply and demand conditions and clears all markets. The competitive equilibrium

conditions are given by the standard zero profit conditions and full employment conditions in each country, and the requirement that national supply of each differentiated product equal aggregate world demand for the product. For brevity, we will consider a model with no intermediate input usage in each country.

In the free trade case, the system of equation is

$$\begin{aligned}
 A_j^t w_j &= p_j & \forall j \\
 A_j y_j &= e_j & \forall j \\
 y_j &= \sum_k x_{jk}
 \end{aligned} \tag{5}$$

A_j is the $h \times n$ matrix of physical input coefficients in the competitive equilibrium for the country concerned and A_j^t is its transpose. p_j is the (column) vectors of product prices whose i 'th element is p_{ij} and y_j is the column vector of the outputs of these products in country j . e_j is the vector of factor endowments for the country and w_j is the vector of (national) prices of these factors. $\sum_k x_{jk}$ are the vector of aggregate demands for the products with x_{jk} the vectors of demands for the outputs of country j from consumers located in country k . Consumer incomes are determined by the factor prices together with factor ownership. Consumer demand for each product in each country depends on incomes and product prices in the country. The $mn+mh+mn$ equations equations in Equation (5) jointly determine the product prices, factor prices and industry outputs in all countries.

In the case of distorted trade, we assume that the distortions take the form of ad valorem tariffs or fixed ad valorem equivalents of non-tariff barriers. For any product which is a member of the i 'th product group, produced in country j , and consumed in country k and subject to some border tariff, the divergence between the pre-tariff and post-tariff prices is

$$p_{ijk} = p_{ij} (1 + t_{ijk}) \tag{6}$$

where t_{ijk} is the tariff on the imports of the product sourced from country j into country k . If tariffs, do not discriminate according to the source of the product, $t_{ijk} = t_{ilk} = t_{ik}$ for all j and k . The competitive equilibrium is now given by Equations (5) and (6).

In both free trade and tariff situations, these conditions are taken to hold as equalities, rather than as inequalities combined with slackness conditions. The latter allow some zero outputs or some unemployed factors. It is assumed there is sufficient regularity in the technology to rule these out. In any case, the Armington Assumption coupled with the assumption that the demand is CES require a strictly positive output of all products in all countries. The outputs and trade pattern of each country are determinate for all dimensions of the model, in free trade and tariff situations alike. This holds because, in the Armington model, countries produce distinct products in each industry and have, in general, distinct technologies. By contrast, in the Heckscher-Ohlin model, outputs and trade are indeterminate when there is free trade, identical technologies, and the number of goods is greater than the number of factors (see, for example, Choi, 2003 and Deardorff, 2005.) There is a possibility of multiple equilibria but multiple equilibria do not seem to arise in computable versions of the model.

3. Properties of the Armington model as a general equilibrium trade model.

The properties of the Armington model are quite different in some respects than those of the Heckscher-Ohlin or the specific factor model with homogeneous outputs. Some of the properties are obvious and others are less so.

An obvious implication of the Armington Assumption is that each country is unspecialised in the products it produces and trades. Indeed, if the sub-utility functions in all countries are CES, each country must produce and trade its national product in every industry. Thus, all countries are completely unspecialised. This overcomes the problem of excessive specialisation in Heckscher-Ohlin models with many goods and factors. Complete unspecialisation is realistic for the level of aggregation of both countries and commodity groups used in cge models.

A second obvious result is that, in free trade situations, the Law of One Price does not hold as it does in the Heckscher-Ohlin model. This is well understood but it needs to be stated precisely because of its role in deriving other properties of the model.

The Law of One Price

As before, p_{ijk} denotes the price of a product of industry i originating in country j and sold in country k . Assume now that there are zero trade costs in the world economy, that is, zero transport costs, tariffs and other government-imposed restrictions on international trade in the products in all countries.

Proposition 1. When there are zero trade costs in the world economy, the Law of One Price holds in the form

$$p_{ijk} = p_{ij\ell}, \quad \forall i, j \text{ and } \ell \neq k$$

but $p_{ijk} \neq p_{i\ell k}, \quad \forall i, k \text{ and } \ell \neq j.$

There is one price for each national product throughout the world, a world price. This result holds because of perfect competition in a completely integrated world market for the good. However, in an Armington model, the outputs of the industry produced in different countries are imperfect substitutes for each other and will, therefore, sell for different prices throughout the world.

By contrast, in a standard Neoclassical model, such as the Heckscher-Ohlin or the specific factor model, it is assumed that the outputs of an industry originating in different countries are perfect substitutes. Assuming again that each industry produces a single output in each country, there is a single worldwide price for the output of an industry. That is,

$$p_{ijk} = p_{ij\ell}, \quad \forall i, j \text{ and } \ell \neq k$$

and $p_{ijk} = p_{i\ell k}, \quad \forall i, k \text{ and } \ell \neq j. \tag{7}$

Dropping the second subscript, $p_{ik} = p_{i\ell}$, for all i and all consuming countries k and ℓ and $\ell \neq k$.

Here p_{ik} denotes the price of the homogeneous product of industry i sold in country k .

One immediate corollary of this proposition is that factor prices are not, in general, equalised across countries in the Armington model when product prices are equalised due to the absence of trade costs.

With zero trade costs, there is no single world price for the outputs of an industry and, hence, there can be no unique mapping of goods prices to factor prices that holds across countries. This result holds even when a condition holds that is sufficient for factor price equalisation in the Heckscher-Ohlin model with non-differentiated products.

Comparative Advantage

In the Armington model, one country cannot have a price advantage in producing one product relative to the prices of other products over other countries because the sets of products produced by any two countries are disjoint. There is no comparative advantage in the Armington model, except in the trivial sense that each country specialises in the production and export of the products which it alone can produce. There is nothing in the model to explain how this differentiation arises.

Yet, national endowments and technologies clearly play a role in determining the pattern of trade in the Armington model, as they do in other Neoclassical models. The homogeneous (undifferentiated) goods of the Heckscher-Ohlin model emerge as the limit case of the Armington model as the elasticities of substitution for each group $\sigma_i \rightarrow +\infty$ for all groups of products. This suggests that national endowments and technologies play a role in determining the pattern of trade even when $\sigma_i < +\infty$.

The key to understanding the pattern of trade lies in the distinction between the gross trade and the net trade in the products of an industry. The gross trade in the products of an industry is the aggregate value of exports and of imports of the products classified in the industry. Net trade is the difference between the aggregate value of these exports and imports. This difference is sometimes called inter-industry trade. The total trade in the products of an industry is the sum of inter-industry trade and intra-industry trade in the products of the industry. That is, for industry i in country j ,

$$X_{ij} + M_{ij} = |X_{ij} - M_{ij}| + \{X_{ij} + M_{ij} - |X_{ij} - M_{ij}|\} \quad (8)$$

X_{ij} and M_{ij} are the value of exports and of imports of the variant(s) of industry i produced by country j and imported by country j respectively. Let z_{ij} and m_{ij} be the quantities of the goods exported and imported respectively. Then $X_{ij} = (p_{ij} z_{ij})$ and $M_{ij} = (\sum_{l \neq j} p_{ijl} m_{ij})$. This is the terminology of intra-industry trade theory.

In a model of intra-industry trade in which there are economies of scale and imperfect competition, Helpman (1981) proved that net or inter-industry trade is determined by factor proportions. He assumed that the technologies were identical across countries, as is done in the proof of the Heckscher-Ohlin Theorem, and added a lot of symmetry among consumers. In his model there are two factors, two goods and two countries. Helpman and Krugman (1985, Part III) applied the same technique to the Krugman love-of-variety form of product differentiation.⁷

A similar result can be proven for the Armington model with dimensions $(m \times n) \times (h \times m)$. The different differentiated commodities in one group can be aggregated in the Armington model if there exist aggregators in both demand and supply with suitable properties and if the aggregation applies to the same groups in both demand and supply. These groups are the groups of products viewed by consumers as a separable group. For producers we can regard them as the products of an "industry". The industry is a device which allows us to view the world economy at two levels, the industry level and the commodity or intra-industry level. At the top level the dimensions have been reduced from the number of elementary commodities to the number of aggregated commodities.

Consider a free trade competitive equilibrium.

On the supply side, no aggregation is possible if the technologies differ across countries. If, however, the technologies for producing each variant of the good produced in each country are all identical, then the aggregator function is simply the production function for each and every product in the group. If this holds, $y_{ij} = \psi_{ij}(e_{ij}) = \psi_i(e_{ij})$. It is now convenient to introduce a notation for these top-level groups. Bold letters will be used to denote aggregated top-level variables or coefficients.

Then, at the top level, the industry production functions are

$$\mathbf{y}_{Ij} = \psi_I(\mathbf{e}_{Ij}) \quad \forall I \quad (9)$$

with $I = 1, \dots, n$ indexing the group of products. Each production function has a dual cost function.

The producer prices will equal unit costs in the industry;

$$\mathbf{p}_{Ij} = \mathbf{c}_{Ij}(w_j) \quad \forall I, j \quad (10)$$

Since the top level products are the same across countries, each country has a comparative advantage in producing some goods.

To make this top-level sub-model isomorphic to the Heckscher-Ohlin model, we have to consider the demand side of the model. The utility function of the consumer in each country is homogeneously separable, as in Equation (1). We can consider that the consumer in each country maximises utility by choosing the top-level quantities, given the prices in the competitive equilibrium. For each product group there is an aggregator function, $v^{ik}(x_{ik})$ in country k . We now have $\mathbf{v}^I(x_{ik})$. This is a quantity index of the i 'th group. Each of these quantity indices has a dual consumer price index

$$\mathbf{p}_{Ik} = \Phi^I(p_{i1k}, \dots, p_{ink}) \quad \forall I \text{ and } k \quad (11)$$

This function is linearly homogeneous in its arguments. As the utility functions have been assumed to be identical across countries, the quantity and price indices are identical across the countries. With free trade, the prices faced by consumers are the same for each product and for each product group. There is no need, therefore, to attach a country of consumption subscript to prices. Thus, the p_{ijk} are identical in all consuming countries and the price index $\mathbf{p}_{Ik} = \mathbf{p}_I$ in all countries. In each consuming country k , the demand for an industry's products is

$$\mathbf{x}_{Ik} = \mathbf{x}_{Ik}(\mathbf{p}_I, w_k) \quad \forall I \text{ and } k \quad (12)$$

These functions are the product of maximising utility at the top level.

Aggregation in demand is consistent in each country in the sense that expenditure on the aggregate quantity at the top level equals the sum of the expenditures on the products in the group in the full

competitive equilibrium, viz. $\mathbf{p}_1 \mathbf{v}^k(x_{ik}) = \sum_j p_{ijk} x_{ijk}$. This is a consequence of homogeneous separability, which is a part of the Armington Assumption.

For aggregation in both demand and supply simultaneously, the units used to measure quantities of goods produced and consumed must be the same and the prices of these industry outputs in Equation (10) must be equal to the prices \mathbf{p}_1 given by Equation (11). For the quantities, the units will be the same if, on the demand side, the consumer regards all products in the group symmetrically; if the utility function is CES, the weights attached to each product in the sub-utility index given by Equation (2), α_{ijk} , must be the same for all j sources of these products. For prices, from Equation (10), $\mathbf{p}_{1j} = \mathbf{p}_{1\ell}$. But $\mathbf{p}_{1j} = p_{ij}$, the price of the one product produced by country j , and $\mathbf{p}_{1\ell} = p_{i\ell}$, the price of the one product produced by country ℓ . Therefore, it must be that $p_{ij} = p_{i\ell}$. One condition that is sufficient for this equality of product prices is factor price equalisation in the model (see below).

There is a symmetric equilibrium in terms of product prices and quantities. This technique of aggregation via the assumption of identical technologies for producing each of the products within a group and across countries and symmetry of demand is in essence the same technique as that used by Helpman (1981) and by Helpman and Krugman (1985) in their models of intra-industry trade with differentiated products of the Helpman-Lancaster type and the Krugman type.

There is still trade in differentiated products under these conditions as consumers prefer a convex combination of products if the sub-utility function is CES or any other strictly quasi-concave function. However, the assumption of identical technologies and products reintroduces indeterminacy of outputs and trade patterns if the number of goods is greater than the number of factors, as in the Heckscher-Ohlin model. In the Armington model, this indeterminacy does not matter, as the case of identical products and technologies is a hypothetical case to disentangle the factors causing inter- and intra-industry trade.

To verify the aggregation is consistent for trade flows, we define the exports or imports of country j at the top level as

$$\mathbf{X}_{1j} \text{ or } \mathbf{M}_{1j} = \mathbf{p}_{1j} \mathbf{y}_{1j} - \mathbf{p}_{1j} \mathbf{x}_{1j}$$

$$\begin{aligned}
&= \mathbf{p}_{ij} y_{ij} - \sum_j \mathbf{p}_{ij} x_{ij} && \text{[by definition of terms]} \\
&= (\mathbf{p}_{ij} y_{ij} - \mathbf{p}_{ij} x_{ij}) - \left(\sum_{l \neq j} \mathbf{p}_{ilj} m_{lj} \right) \text{ [since } x_{ij} = m_{ij} \text{ for } j \neq l \text{]} \\
&= X_{ij} - M_{ij} && (14)
\end{aligned}$$

The absolute value of this term is the value of inter-industry trade as defined in Equation (8). That is, because both consumption and production of the products can be consistently aggregated, exports or imports (= excess demand) can also be consistently aggregated. Inter-industry trade in the full competitive equilibrium is equal to the excess demand for the good at the top level of the model.

We have now all of the elements of a top-level competitive equilibrium. The conditions for a top-level equilibrium are

$$\begin{aligned}
\mathbf{A}_j^t \mathbf{w}_j &= \mathbf{p}_j && \forall j \\
\mathbf{A}_j \mathbf{y}_j &= \mathbf{e}_j && \forall j \\
\sum_j \mathbf{y}_j &= \sum_k \mathbf{x}_k && (13)
\end{aligned}$$

where $\mathbf{p}_j = (\mathbf{p}_{1j}, \dots, \mathbf{p}_{lj}, \dots, \mathbf{p}_{nj})$ whose elements are now regarded as the prices of the industry aggregates and the input shares are defined for these outputs, \mathbf{y}_j and \mathbf{x}_k are the vectors of industry outputs and demands in countries j and k respectively. The first two subsets of conditions for the top level have the same form as those in Equation (5) but that the outputs and prices are now those of the industry rather than of the individual product variant produced in each country. The third is now a statement that, in each goods market, the world supply equals the world demand.

Together, this top-level competitive equilibrium is identical to that of a Heckscher-Ohlin model of the same dimensions, namely n goods, h factors and m countries. Therefore, it has the same properties as a Heckscher-Ohlin model of the same dimensions. In particular, the inter-group trade pattern is determined by factor abundance.

Proposition 2. If there are zero trade costs and the conditions necessary for consistent aggregation are satisfied on both the demand and supply sides of the model, factor abundance determine the trade in top-level goods.

If we make the additional assumption that the top-level utility function is homothetic, then the Heckscher-Ohlin Theorem applies exactly as for a Heckscher-Ohlin model of the same dimensions. For example, if the dimensions of the model are $2 \times 2 \times 2$, as in the standard textbook version of the Heckscher-Ohlin model, and we assume that the top-level utility function is homothetic, then the quantity version of the Heckscher-Ohlin Theorem holds: each country has (inter-industry) exports of the good which uses intensively the factor with which it is well endowed. Furthermore, if the factor endowment ratios of the two countries as well as the technologies are identical, there is no inter-industry trade (but there is still intra-industry trade). If, instead, the dimensions of the top level of the model are $h \times m \times n$ and we assume that the top-level utility function is homothetic, then the factor content form of the factor proportions theorem holds as in Vanek (1968): each country is a net exporter/importer of the services of the factor with which it is well/poorly endowed relative to the world.

If nationally differentiated intermediate inputs and international trade in these inputs are introduced, aggregation of the demand for and supply of these intermediate inputs can proceed in the same manner. Net or inter-industry trade is now a combination of trade in final products and trade in intermediate inputs, but there is a top-level representation of the model which is identical to a Heckscher-Ohlin model of the same dimensions.

Thus, we use the Heckscher-Ohlin Theorem to explain inter-industry trade. We can say that, under these conditions, there is comparative advantage in the groups of products, though not in the individual products themselves. Trade in the elementary products, intra-industry trade, is determined by national product differentiation and preferences defined over these differentiated products.

Although the conditions required for this result are extremely stringent in the Armington model, this result gives a fundamental insight into the determination of the pattern of trade in the model. This

result is not surprising when we view the Armington model as if it starts with a standard Neoclassical model with homogeneous outputs in each industry and then add Armington differentiation.

In the Armington model there are three causes of trade – differences in endowments, differences in technology and product differentiation. If both the endowments and the technologies are identical across all countries, product differentiation alone is sufficient to cause trade. By comparison, under these conditions, there would be no trade in a Heckscher-Ohlin model. If the competitive equilibrium cannot be aggregated into a top level competitive equilibrium, factor proportions and differences in technologies and product differentiation will all interact to determine jointly the pattern of trade in the competitive equilibrium.

We can obtain a second proposition concerning the pattern of trade that is related to the Armington elasticities.

Proposition 3. Consider an initial competitive equilibrium for the world economy. Assume that there are zero trade costs and that the conditions sufficient for consistent aggregation are satisfied on both the demand and supply sides of the model. If the elasticities of substitution between home and foreign-produced goods increase, then the proportion of intra-industry trade decreases. As the $\sigma_i \rightarrow +\infty$ for all groups of products, this proportion tends to zero.

In the initial competitive equilibrium, inter-industry trade is determined by the top-level conditions. These do not change as the lower-level elasticities of substitution change. Suppose initially there are only two countries, the home country and the foreign country. If the second-level elasticity of substitution increases, each indifference curve becomes flatter (see Chipman, 1966, p. 58). The combination of home and foreign products chosen in the initial equilibrium is no longer optimal. As the indifference curve at the initial equilibrium has become flatter, the consumer in each country will move towards the axes. The consumer will buy more of the product that is cheaper. The consumer will tend to buy from a single source. In contrast, a low elasticity of substitution forces a more even mixture of products.

If there is more than one foreign country, suppose the functions at both the second and third levels of Equation (4) are CES. Again intra-industry trade decreases from the level observed in the initial competitive equilibrium. At the third or lowest level, the same kind of substitution occurs.

Consumers will buy more of the foreign products of each industry from the cheapest foreign source.

This proposition confirms the intuitive view that it should hold. The proposition extends to a symmetric equilibrium with tariffs. It may hold more generally.

Factor Price Equalisation

Although factor price equalisation does not hold generally, we can use the top-level aggregation condition to find a condition under which factor prices are equalised.

Proposition 4. Factor prices are equalised with zero trade costs if the conditions necessary for consistent aggregation are satisfied on both the demand and supply sides of the model, the number of goods equals the number of factors, and the country endowments lie in the same cone of diversification.

Under the first set of conditions, as noted in the previous section, the products in the model may be aggregated and we may write the model at the top level as if it is a lower-dimensional Heckscher-Ohlin model. The other conditions reproduce a combination of assumptions that is sufficient for factor price equalisation in the Heckscher-Ohlin model. These conditions are extremely unrealistic as they restrict the technology and the endowments and rule out specific factors in the model.

Gains from trade

Cge models using the Armington Assumption have proven a useful vehicle for exploring gains from greater trade due to the reduction in trade barriers. However, the nature of gains from trade in an Armington model differs from that in a Heckscher-Ohlin (or specific factor) model.

In an Armington model, there is no comparative advantage in products, as noted in the previous section. Consequently, gains from trade due to greater specialisation in products in which a country has a comparative advantage and greater importing of products in which it has a comparative

disadvantage cannot arise. These are the standard gains, on the supply side, in a Heckscher-Ohlin or a Classical model of international trade. At the level of industries, opening up international trade may induce movements of resources between industries but the price of a “good” produced by an industry is an index of prices of products produced by many countries and the effects of one country’s market opening is muted. On the demand side, an Armington model has additional gains from consuming a greater variety of the products of each industry. These differences have important consequences.

To analyse these differences, consider the set of conditions for a free trade competitive equilibrium given by Equation (5). In the first two subsets of equations, we can regard the supply as a function of domestic producer prices. These first two subsystems of equations can be solved to give the supply and factor price functions

$$y_j = y_j(p_j, e_j) \quad \forall j$$

$$\text{and} \quad w_j = w_j(p_j, e_j) \quad \forall j \quad (15)$$

p_j and w_j are the vectors of product and factor prices in country j . Substituting these equations in Equation (5) gives

$$y_j(p_j, e_j) = \sum_k x_{jk}(p_k, e_j) \quad (16)$$

These $m \times n$ equations are a reduced form of the competitive equilibrium conditions. The solution gives the set of competitive equilibrium prices, noting that $p_k = p_j$, the vector of world prices. This holds for any number of goods and factors in the Armington model.

The solution holds for a tariff situation as well as for free trade. In this case, the prices on the left-hand side of Equation (16) are the pre-tariff prices in the supplying countries and those on the right-hand side are the post-tariff prices in the consuming countries. These prices are linked, for each product, by Equation (6). Substituting Equation (6) in Equation (16), gives the reduced form equations. The solution to these equations give the pre-tariff international prices for each product, p_j .

Proposition 5 The competitive equilibrium for the Armington model can be written in a reduced form that involves equations representing only the market clearing conditions as functions only of

international product prices. In this system of equations, there is one equation for each national product in each product group.

One cannot obtain the same reduced form for the Heckscher-Ohlin model. When the products produced by an industry in all of the countries are homogeneous, not Armington-differentiated, the goods market clearing conditions are

$$\sum_j y_{jk} = \sum_k x_{jk} \quad (17)$$

There are only n such equations compared to $m \times n$ in the Armington model. The conditions for a competitive equilibrium are given by Equation (17) and the first two subsets of equations in Equation (5). The zero profit and full employment conditions of Equation (5) still hold for each country, although the world price of good i is now equal across countries. One can still use the first two subsets of zero profit conditions to eliminate the factor prices as variables but the reduced form is now

$$\sum_j y_{jk}(p_j, e_j) = \sum_k x_{jk}(p_k, e_j) \quad (18)$$

In this equation p_k is an n -vector of goods prices in the consuming country k . If there is not free trade, the p_k and p_j are linked by Equation (6). There are only n equations.

This difference in the market clearing equations gives national producers more monopoly power compared to the Heckscher-Ohlin model, in both free trade and tariff situations⁷. In the Heckscher-Ohlin model, each country is, strictly speaking, large in the sense that its demand and supply affect prices of the goods it trades. Consequently, unilateral reductions in trade barriers by one country lead to a worsening in the terms of trade. But this monopoly power is constrained because countries sell homogeneous goods in world markets in competition with sellers from other countries. By contrast, in the Armington model, there are distinct prices for the products of each country because of product differentiation, and the elasticities of substitution between national products of the same group are less than infinite, usually much less. The Law of One Price does not hold in the form given by the Heckscher-Ohlin model. Firms no longer equate their unit costs to world prices, as in the Heckscher-

Ohlin model. Instead, the national firms' unit costs are now the world prices for their products. They are price makers, not price takers. The assumption that each country is the sole supplier of the products it supplies to the world markets gives it greater monopoly power.

This greater monopoly power affects the comparative statics of the model. Consider a tariff situation. Now let the market conditions be improved in some country or countries by a reduction in these tariff rate(s) in some group. Adjustment in the world markets occurs on the supply sides in each country.

In the Heckscher-Ohlin model, there is greater specialisation in the goods in which each country has a comparative advantage and associated gains from trade. The primary effect is a resource reallocation between industries in each trading country. The price adjusts simultaneously with this resource reallocation in order to clear the market. In an Armington model, by contrast, the prices adjust to clear the market for each product. This price adjustment is the primary mechanism for reaching a new equilibrium. The adjustment of output quantities is muted by the lack of direct competition between national producers.

We can see these results by examining the equations for any one of the product groups. For the Armington model, the system of equations is

$$y_{ij}(p_{ij}, e_j) = \sum_k x_{ijk}(p_{i1k}, \dots, p_{ink}, \dots, e_j) \quad \forall j \quad (19)$$

whereas the Heckscher-Ohlin model has the one equation

$$\sum_j y_{ij}(p_{ij}, e_j) = \sum_k x_{ijk}(p_{ik}, \dots, e_j) \quad (20)$$

In order to focus on the direct changes in the market which is disturbed, we ignore the adjustments in demand and supply across goods markets that occur in the general equilibrium solution. The dots among the arguments of the function on the right-hand side of the equations represent the prices of other inter-group products being held constant.

We now differentiate totally Equation (20) with respect to the changes in the prices in the market for the i 'th good,

$$\sum_j \eta_{ij} \hat{p}_i = \sum_j \varepsilon_{ij} \hat{p}_j \quad (22)$$

The changes in all variables have been expressed in terms of proportionate changes. η_{ij} and ε_{ij} are the elasticity of supply in response to the change in the home country's product price and the elasticity of demand with respect to the change in the price of the product sourced from country j and sold in country k. The left-hand side represents the response in each producing country to the change in the price of good i while the right hand side represents the response in each consuming country, holding constant the prices of goods produced by other industries.

In the Armington model, from Equation (19), the differential equation is

$$\eta_{ij} \hat{p}_i = \sum_k \sum_j \varepsilon_{ijk} \hat{p}_{ijk} \quad (23)$$

η_{ij} is again the elasticity of supply in response to the change in the home country's product price and ε_{ijk} is the elasticity of demand with respect to the change in the price of the distinct national product sourced from country j and sold in country k.

Comparing Equation (23) with (22), we see that the supply response in the Armington model lacks the shift in comparative advantage that is present in the Heckscher-Ohlin model⁹ but, on the demand side, it has the additional response as consumers shift demand among the close substitute products within the group as their market prices change.

This feature of the model explains why the observed changes in output are dominated by changes in demand in Armington models. Changes in market access do not lead typically in Armington models to substantial changes in national outputs. By contrast, in the Heckscher-Ohlin model, Deardorff (2005) shows that the many-goods many-factor many-country version of the model is "hyper-sensitive" to changes in tariffs or other trade costs that disturb the equilibrium. In particular, small changes in trade costs can cause production of goods in a country to appear or disappear, and this behaviour is discontinuous in the level of the trade costs. The Armington model has resolved this

problem but it has gone to the opposite extreme of having no comparative advantage adjustments. On the other hand, it has added adjustment by consumers to the changes in the prices of competing products.

This feature also explains the large terms of trade effects of tariff changes which have been often observed in Armington models, even when the country concerned accounts for a very small share of world production and trade. These larger price changes are necessary to clear the market when the quantities produced and consumed change as in Equation (23).

This effect is due to the Armington Assumption itself, not to the choice of Armington elasticities or other features of the paramaterisation of the model. These Armington elasticities do play a secondary role in determining the magnitude of the terms of trade effects as the cross-price elasticities of demand in Equation (23) are increasing functions of the Armington elasticities. There is a considerable literature on the effects of these elasticities on the magnitude of the terms of trade effects. (See particularly, Brown, 1987 and, for a model with intermediate inputs, Saito, 2004. Zhang, 2006 provides further discussion of the roles of the Armington Assumption and the Armington elasticities in determining terms of trade effects point.

5. General comments on the Armington model

The main builders of the cge models based on the Armington Assumption seem to regard the Assumption as a satisfactory approximation to the product differentiation observed in the real world. In discussing his suite of models, Whalley (1985, p. 38) states that “Though not wholly satisfactory (in that cross-hauling is accommodated rather than explained), the use of the Armington assumption in models here has had some benefits...”. In discussing the nature of the GTAP model, Hertel and Tsigas (1997, p. 41) conclude that “In sum, although we are not particularly happy with the Armington specification, it *does* permit us to explain cross-hauling of similar products and to track bilateral trade flows.” The Armington model does introduce product differentiation as a cause of international trade, in addition to factor endowments and differences in technologies, and it allows intra-industry trade. This was a big advance in trade theory.

Most critical attention has focused on the large terms of trade effects of tariff changes in an Armington model. This debate has been about the values of the elasticity of substitution parameters at various levels.

There is, however, very limited recognition of the peculiarity of the Armington Assumption and its effects on policy analyses from models incorporating this assumption. We have seen that the Armington model has a particular form of national horizontal differentiation of products where each national industry produces a single differentiated product whose distinctiveness is exogenous and unexplained.

Fixity of the number of products does not allow any changes in product varieties on the global markets as tariffs, technologies and other market conditions change. The standard Neoclassical models also has a fixed number of commodities. However, this fixity is more serious in an Armington Model than in a Neoclassical model. In an Armington model, the focus is on differentiation of products within a group. Changes in the composition of the group will occur when relative prices and incomes change with any shock to the general equilibrium. One part of this change is that the numbers of products produced and consumed in each group will normally change. This intra-industry variation is ruled out by the Armington Assumption. Hertel and Tsigas in Hertel (1997, p. 41) acknowledges fixity of the number of varieties as a “fundamental critique” of the Armington Assumption.

Armington differentiation has other more subtle effects. It removes comparative advantage in the trade of products. Compared to the Heckscher-Ohlin model, quantity adjustments by producers to policy shocks are smaller while the terms of trade effects are greater. This is a consequence of the Armington specification rather than the choice of elasticities of substitution.

If the national product differentiation which is built in to every market in an Armington cge model exaggerates the amount of monopoly power which countries really do have, as some believe, the Armington Assumption will bias the results of policy simulations based on these models. For example, simulations of the effects of improved market access will understate the aggregate world gains from trade, because of the low supply responses; and will distribute these gains differently,

smaller gains or even losses going to larger countries which suffer greater deterioration in their terms of trade. Gains from greater trade are understated too because fixity of the number and nature of the differentiated products cannot capture the welfare effects due to the introduction of new product varieties for both final and intermediate goods as tariffs change. These features also understate the production sector adjustments in a model.⁸ The absence of comparative advantage mutes the supply responses. In particular, it is not possible in an Armington model with CES preferences for changes in tariff rates to cause an industry to close down or to start up. Similarly, it is not possible for consumers to stop or start buying particular products. In interpreting the results of simulations from Armington models, all users should be aware of the properties of the model and their implications for modelling policy changes.

These features raise the question of whether they are due to the specific form of product differentiation adopted by Armington or hold more generally for trade models with horizontal product differentiation. Each model of trade with some type of differentiated products has, of course, unique properties. The task here is to discern commonalities among these models.

A form of the Law of One Price in Proposition 1 and the general absence of factor price equalisation will hold for all forms of horizontally differentiated products. Variants of Proposition 2 relating to factor proportions explanations of inter-industry trade and Proposition 4 relating to factor price equalisation hold for both the “love of variety” or the “preferred product” forms of horizontal differentiation (Helpman and Krugman, 1985, Part III) and for a model combining a continuum version of Heckscher-Ohlin with Krugman differentiation (Romalis, 2004). Proposition 3 relating to intra-industry trade may hold generally. These remarks indicate that propositions concerning the theory of trade in differentiated product encompassing the “love of variety” and the “preferred product” types developed by Helpman and Krugman can be extended to Armington-differentiated products. These models share common structural elements. The “love of variety” and the “preferred product” types followed Armington in introducing weak separability into the utility function to define the product groups, and in all three models identical technologies across countries and products and symmetry in demand have been used to derive propositions concerning factor proportions and factor

price equalisation. These common features may extend to other types of horizontal product differentiation.

However, Proposition 5 on the nature of the competitive equilibrium above depends crucially on Armington-type national differentiation of the products. Similarly, the fixity in the number of products produced and consumed, give this model some distinctive features.

Ultimately, the choice between forms of product differentiation is an empirical issue. It may be that the homogenisation of tastes associated with globalisation and the development of products such as the global car has reduced the extent of national product differentiation in international trade.

FOOTNOTES

*

1. This ignores internal transport costs and other factors that may cause a homogeneous commodity to be both exported and imported from one nation.
2. The first to have done this in a fully specified cge model of the world economy with several countries were John Whalley in a series of papers starting from the mid-1970s (see Whalley, 1985 and references therein), Miller and Spencer (1977) who built a model to analyse the effects of UK entry into the EEC, and Lloyd (1979a, 1979b) who used the Whalley model to study intra-industry trade. They were followed soon after by Deardorff and Stern (1981) in the first version of the Michigan Model and other early multi-country models using the two-stage CES function (for a list, see Shoven and Whalley, 1984, Table 8).
3. In some models a higher stage involving the allocation of the household's budget between consumption and savings.
4. For example, the final goods sourced from each country in the model are essential in that, in every country, a strictly positive quantity of each product is demanded at anything less than an infinite (relative) price for the product. Geometrically, the isoquants in a CES indifference map do not intersect the axes or intersect them asymptotically. Essentiality has the important consequence that an economy cannot be closed in a world with finite tariff rates. The usual measure of gains from trade is the elevated level of welfare with free trade, or with some trade in a tariff-distorted equilibrium, compared to the level with no trade. The standard gains from trade propositions do not hold, strictly speaking, in an Armington model. However, one can get arbitrarily close to no trade by continuously raising tariff rates.

5. Indeed, it should be remembered that Armington was not building a model of the world economy. He specified his demand structure in order to understand the changing composition of imports.
6. The Armington demand structure has sometimes been combined with increasing returns to scale and/or imperfect competition. However, the simultaneous introduction of either of these features itself substantially changes the properties of the model and it is not, therefore, simple to determine the net effect of introducing the Armington structure.
7. It has considerable advantages in computing the competitive equilibrium but we do not pursue this aspect here.
8. This aspect is exacerbated by the high level of aggregation of goods in most cge models.

REFERENCES

- Armington, P.S., "A Theory of Demand for Products Distinguished by Place of Production,"
International Monetary Fund Staff Papers 16 (1969): 159-176.
- Brown, D. K., "Tariffs, the Terms of Trade, and National Product Differentiations," *Journal of Policy Modeling* 9(1987) : 503-526.
- Chipman, J. S., "A Survey of the Theory of International Trade: Part 3, The Modern Theory,"
Econometrica 34 (1966): 18-76.
- Choi, E. K. "Implications of Many Industries in the Heckscher-Ohlin Model" in E. K. Choi and J. Harrigan (eds.), *Handbook of International Trade*, Malden, MA and Oxford: Blackwell (2003).
- Deardorff, A.V., "The Heckscher-Ohlin Model: Features, Flaws and Fixes", Nottingham Lectures (2005), available at www.gep.org.uk.
- Deardorff, A.V. and Stern, R. M., "A Disaggregated Model of World Production and Trade: An Estimate of the Impact of the Tokyo Round," *Journal of Policy Modeling* 3 (1981): 127-52
- Dixon, P. B., Parmenter, B. R. Sutton, J. and Vincent, D. P. *ORANI: A Multisectoral Model of the Australian Economy*, Amsterdam: North Holland (1982).
- Dixon, P. B. and Rimmer, M. T., *Dynamic General Equilibrium Modelling for Forecasting and Policy*, Amsterdam: Elsevier (2002).
- Head, K. and Ries, J., "Increasing Returns versus National Product Differentiation as an Explanation for the Pattern of U.S.-Canada Trade", *American Economic Review*, 91 (2001): 858-76.
- Helpman, E., "International Trade in the Presence of Product Differentiation, Economies of Scale and Monopolistic Competition: a Chamberlin-Heckscher-Ohlin Approach," *Journal of International Economics* 11 (1981): 305-40.

- Helpman, E. and Krugman, P. R., *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition and the International Economy*, Cambridge, Mass: The MIT Press (1985).
- Hertel, T., *Global Trade Analysis: Modeling and Applications*, Cambridge and New York: Cambridge University Press (1996).
- Hertel, T. and Tsigas, M. E., Structure of GTAP, in : Hertel, T. (Ed.) , *Global Trade Analysis: Modeling and Applications*, Cambridge and New York: Cambridge University Press (1997).
- Krugman, P. R., “Increasing Returns, Monopolistic Competition and International Trade,” *Journal of International Economics* 9 (1979): 469-79.
- Krugman, P. R., “Scale Economies, Product Differentiation and the Pattern of Trade,” *American Economic Review* 70 (1980): 950-59.
- Lancaster, K. “Intra-industry Trade under Perfect Monopolistic Competition,” *Journal of International Economics* 19 (1979): 1511-75.
- Lloyd, P. J., “Intra-industry Trade, Lowering trade Barriers and Gains from Trade” in: Giersch, H. (Ed.), *On the Economics of Intra-industry Trade*, Tübingen: J. C. B. Mohr (1979a).
- Lloyd, P. J., “Tax Distortions and the Rate of Economic Growth: A Numerical General Equilibrium Analysis,” *Keio Economic Studies* 15 (1979b), 51-
- Miller, M. H. and Spencer, J. E., “The Static Effects of the UK Joining the EEC: A General Equilibrium Approach,” *Review of Economic Studies* 44 (1977): 71- 93.
- Romalis, J., “Factor Proportions and the Structure of Commodity Trade”, *American Economic Review*, 94 (2004): 67-97.
- Saito, M., “Armington Elasticities in Intermediate Inputs Trade: A Problem in Using Multilateral Trade Data,” *Canadian Journal of Economics* 37 (2004): 1097-1117.

- Sato, K., "A Two-level Constant-elasticity-of-substitution Production Function," *Review of Economic Studies* 34 (1968): 201-218.
- Shoven, J. B. and Whalley, J., "On the Computation of a Competitive Equilibrium on International Markets with Tariffs", *Journal of International Economics* 4 (1974), 139-61.
- Shoven, J. B. and Whalley, J., "Applied General Equilibrium Models of Taxation and International Trade," *Journal of Economic Literature* 22 (1984), 1007-1051.
- Vanek, J., "The Factor Proportions Theory: The n-factor Case," *Kyklos* 78(1968): 749-56.
- Whalley, J., *Trade Liberalization among Major World Trading Areas*, Cambridge, Mass: J. C. B. Mohr (1985).
- Zhang, X., *Armington Elasticities and Terms of Trade Effects in Global CGE Models*. Staff Working Paper, Melbourne: Productivity Commission (2006).